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Some Interesting Properties of Scattering Matrix of Passive Microwave Devices

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Outline

Abstract

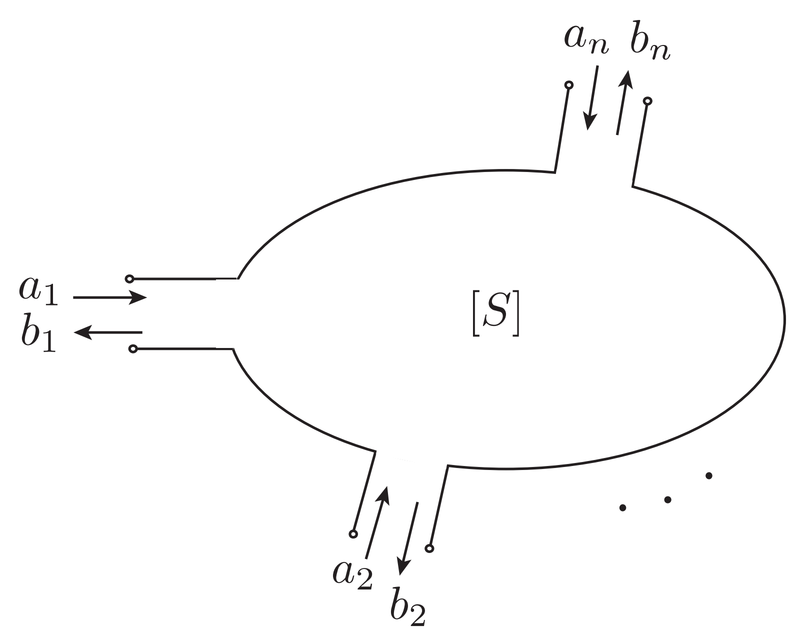
Three useful properties are shown that characterize the dissipative loss and the corresponding efficiency of a multiport, passive microwave network. Elementary examples are considered that involve both reciprocal and non-reciprocal networks. When applied to the equal-split, matched, 3-port resistive divider, they recover the known fact that the device is 50% efficient. The relations yield the new result that the efficiency of a 3-port Wilkinson power divider is 2/3 *on the average*. It is further shown that the Wilkinson power divider belongs to a class of most efficient, matched, reciprocal 3-port networks that are constrained to provide maximum isolation at the output ports.

Keywords: Efficiency, scattering matrix, passive microwave networks, dissipation, hypersphere, eigenanalysis

Goal of Study

Q. Is it possible to define a single metric for a passive *n*-port RF network in terms of its scattering matrix so that various devices could be compared independent of port excitations?

Passive n-Port Network



$$\begin{aligned} \mathbf{b} &= S\mathbf{a}, \quad |S_{ij}| \leq 1,\ i,j = 1,\ldots,n \\ \mathbf{a} &= [a_1, a_2, \ldots, a_n]',\ \mathbf{b} = [b_1, b_2, \ldots, b_n]' \end{aligned}$$

Dissipated Power Metrics

- Power Loss: $P_\ell = \mathbf{a}^\dagger \mathbf{a} - \mathbf{b}^\dagger \mathbf{b}$
- Fractional Power Lost F_ℓ

$$F_\ell = \frac{P_\ell}{\mathbf{a}^\dagger \mathbf{a}} = \frac{\mathbf{a}^\dagger (I - S^\dagger S)\mathbf{a}}{\mathbf{a}^\dagger \mathbf{a}} =: \frac{\mathbf{a}^\dagger H \mathbf{a}}{\mathbf{a}^\dagger \mathbf{a}} \geq 0$$

- Dissipation Matrix $H = I - S^\dagger S$
  - Eigenpair: $(\lambda_i, \mathbf{e}_i),\ H\mathbf{e}_i = \lambda_i \mathbf{e}_i,\ \mathbf{e}_i^\dagger \mathbf{e}_j = \delta_i^j,\ 0 \leq \lambda_i \leq 1$
- Efficiency η_ℓ

$$\eta_\ell = 1 - F_\ell$$

Arbitrary Excitation

$$\mathbf{a} = \sum_{i=1}^{n} \mu_i \mathbf{e}_i$$

μ_i real:

$$\mathbf{a}^\dagger \mathbf{a} = 1 \implies \sum_{i=1}^{n} \mu_i^2 = 1\,.$$

$$\text{Fractional Loss } F_\ell = \sum_{i=1}^{n} \mu_i^2 \lambda_i < 1$$

$$\text{Efficiency } \eta_\ell = \sum_{i=1}^{n} \mu_i^2 (1 - \lambda_i) < 1$$

Averaged Quantities Enable Device Comparisons

μ_i = constrained uniformly distributed RVs on an n-dimensional unit hypersphere | $\sum_{i=1}^{n} \mu_i^2 = 1$

$$\begin{aligned} \text{Average fractional loss } \bar{F}_\ell &= \langle F_\ell \rangle = \sum_{i=1}^{n} \langle \mu_i^2 \rangle \lambda_i \\ \text{Average Efficiency } \bar{\eta}_\ell &= \langle \eta_\ell \rangle = 1 - \bar{F}_\ell \\ \langle \mu_i^2 \rangle &= \frac{1}{n} \end{aligned}$$

Properties

Property (1)

The average fractional loss, $\bar{F}_\ell$, of an n-port microwave network characterized by the scattering matrix S with the corresponding dissipation matrix $H = I - S^\dagger S$ is equal to

$$\bar{F}_\ell = \frac{1}{n}\sum_{i=1}^{n}\lambda_i = \frac{1}{n}\mathrm{Tr}(H) \quad (1)$$

where $\mathrm{Tr}(H)$ denotes the trace of the matrix H.

Properties...

Property (2)

The average efficiency $\bar{\eta}_\ell$ of a passive n-port network is equal to

$$\bar{\eta}_\ell = \frac{1}{n}\mathrm{Tr}(S^\dagger S) = \frac{1}{n}\sum_{i,j=1}^{n} |S_{ij}|^2 = \frac{1}{n}\|S\|_F^2, \tag{2}$$

where $\|S\|_F$ denotes the Frobenius norm of the matrix S.

Properties...

$(H-\lambda I)\mathbf{e}=\mathbf{0} \implies (I-S^\dagger S-\lambda I)\mathbf{e}=\mathbf{0} \implies (S^\dagger S)\mathbf{e}=\mathbf{0}$ when $\lambda=1$.
Non-trivial solution requires $\det(S^\dagger S)=0 \implies \det(S)=0$:

Property (3)

If the scattering matrix of a network with n-available ports is singular, then $\lambda=1$ will be an eigenvalue of the dissipation matrix H, implying that $\bar{F}_\ell \geq 1/n$ and the corresponding average efficiency $\bar{\eta}_\ell \leq (n-1)/n$.

Example-1: Three-Port Resistive Power Divider

$$S = \begin{bmatrix} 0 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix}; \quad H = \begin{bmatrix} \frac{1}{2} & -\frac{1}{4} & -\frac{1}{4} \\ -\frac{1}{4} & \frac{1}{2} & -\frac{1}{4} \\ -\frac{1}{4} & -\frac{1}{4} & \frac{1}{2} \end{bmatrix}.$$

S is non-singular, $\lambda = 1$ is *not* an eigenvalue of H.

$\bar{F}_\ell = 0.5$ and $\bar{\eta}_\ell = 0.5$

(a matched resistive power divider is only 50% efficient).

Example-2: Three-Port Non-ideal Circulator

Signal flow: Port 1$\rightarrow$Port 2$\rightarrow$Port 3

Return Loss = $-20\log\alpha$; Insertion Loss = $-20\log\gamma$; Isolation =$-20\log\beta$

$$S = \begin{bmatrix} \alpha e^{j\psi} & \beta e^{j\theta} & \gamma e^{j\phi} \\ \gamma e^{j\phi} & \alpha e^{j\psi} & \beta e^{j\theta} \\ \beta e^{j\theta} & \gamma e^{j\phi} & \alpha e^{j\psi} \end{bmatrix}$$

$\alpha^2 + \beta^2 + \gamma^2 \leq 1$ ($\because H \geq 0$). $\mathrm{Tr}(H) = 3[1 - (\alpha^2 + \beta^2 + \gamma^2)]$

$$\bar{\eta}_\ell = \left(\alpha^2 + \beta^2 + \gamma^2\right) \leq 1$$

Eg. RL = 10 dB, IL = 3 dB, Is = 20 dB

$\implies \alpha = 1/\sqrt{10},\ \beta = 1/10,\ \gamma = 1/\sqrt{2},\ \bar{\eta}_\ell = 0.61.$

Example-3: 3-Port Wilkinson Power Divider

$$S = \begin{bmatrix} 0 & \frac{-j}{\sqrt{2}} & \frac{-j}{\sqrt{2}} \\ \frac{-j}{\sqrt{2}} & 0 & 0 \\ \frac{-j}{\sqrt{2}} & 0 & 0 \end{bmatrix}; \quad H = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \frac{1}{2} & -\frac{1}{2} \\ 0 & -\frac{1}{2} & \frac{1}{2} \end{bmatrix}.$$

S matrix is singular ($\lambda = 1$ is an eigenvalue of H)

$\bar{F}_\ell = 1/3$; $\bar{\eta}_\ell = 2/3$ (achieves upper limit by Property (3))

Most Efficient Equal Split Three-Port Power Divider

Reciprocal, matched, network with maximum isolation between ports-2 and 3 when fed at port-1

$$S = \begin{bmatrix} 0 & s & s \\ s & 0 & 0 \\ s & 0 & 0 \end{bmatrix}; \quad H = \begin{bmatrix} 1-2|s|^2 & 0 & 0 \\ 0 & 1-|s|^2 & -|s|^2 \\ 0 & -|s|^2 & 1-|s|^2 \end{bmatrix}, \quad |s| < 1$$

S is singular $\implies \bar{\eta} \leq 2/3$ from Property (3)).

$\lambda_i = 1 - 2|s^2|, 1 - 2|s|^2, 1 \implies |s| \leq 1/\sqrt{2}$.

Average efficiency $\bar{\eta} = 4|s|^2/3$.

$|s| = 1/\sqrt{2}$ yields a maximum efficiency of $\bar{\eta} = 2/3$. (Wilkinson power divider, quadrature or 180^o hybrid with one port match loaded are examples)

Conclusions

- Equations (1) and (2) have been established that characterize the average fractional loss and average efficiency of any *n*-port passive network, independent of the excitation.
- Property-3 has been deduced that provides an upper bound to the efficiency of a passive device that has a singular scattering matrix.
- Several examples (resistive power divider, a non-ideal circulator and the Wilkinson power divider) have been considered to demonstrate the utility of the formulas in making device comparisons.
- Wilkinson power divider belongs to the class of most efficient, 3-port reciprocal devices that has the requirement of being matched at all the ports while providing maximum isolation between the output ports.

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Author's Bio

Ramakrishna Janaswamy is a Professor in the Department of Electrical and Computer Engineering, University of Massachusetts, Amherst. He received his Ph.D. degree in electrical engineering in 1986 from the University of Massachusetts, Amherst, the Master's degree in microwave and radar engineering from IIT-Kharagpur, India in 1983 and the Bachelor's degree in electronics and communications engineering from REC-Warangal, India in 1981. From August 1986 to May 1987, he was an Assistant Professor of electrical engineering at Wilkes University, Wilkes Barre, PA. From August 1987-August 2001 he was on the faculty of the Department of Electrical and Computer Engineering, Naval Postgraduate School, Monterey, CA. He was a visiting researcher at the Center for PersonKommunikation, Aalborg, Denmark from September 1997 to June 1998. His research interests are in theoretical/computational electromagnetics, radiowave propagation, antenna theory and design, and wireless communications.

Rama Janaswamy is a Fellow of IEEE and was the recipient of the R. W. P. King Prize Paper Award of the IEEE Transactions on Antennas and Propagation in 1995. He received the IEEE 3rd Millennium Medal in 2000 for his "Outstanding Contributions" to the Santa Clara Valley Section. He is serving/has served as an Associate Editor of (i) IEEE Transactions on Antennas and Propagation, (ii) IET Electronics Letters, (iii) IETE Technical Review (India), (iv) IEEE Transactions on Vehicular Technology and (v) AGU Radio Science. Since July 2013, he has also been serving as a member of the IEEE Antennas and Wave Propagation Standards Committee. He is the author of the book Radiowave Propagation and Smart Antennas for Wireless Communications, Kluwer Academic Publishers, November 2000 and is a contributing author in and a contributing author in Handbook of Antennas in Wireless Communications, L. Godara (Ed.),CRC Press, August 2001 and Encyclopedia